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**METHOD OF REFLECTIONS FOR THE KLEIN–GORDON EQUATION**

**Abstract.** Using the method of reflections, the solutions of the first and second mixed problem for the homogenous Klein–Gordon equation in a quarter plane and of the first mixed problem for the homogenous Klein–Gordon equation in a half-strip are written out in an explicit analytical form. The Cauchy conditions of these problems are inhomogeneous, but the Dirichlet boundary condition (or the Neumann boundary condition) is homogeneous. Conditions are formulated, under which the solutions to these problems are classical.

**Keywords:** Klein–Gordon equation, method of reflections, mixed problem, classical solution

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**МЕТОД ОТРАЖЕНИЙ ДЛЯ УРАВНЕНИЯ КЛЕЙНА–ГОРДОНА**

**Аннотация.** Методом отражений в явном аналитическом виде выписаны решения первой и второй смешанных задач для однородного уравнения Клейна–Гордона в четверти плоскости и первой смешанной задачи для однородного уравнения Клейна–Гордона в полуполосе с неоднородными условиями Коши и однородным условием Дирихле (или условием Неймана). Сформулированы условия, при которых решения данных задач являются классическими.

**Ключевые слова:** уравнение Клейна–Гордона, метод отражений, смешанная задача, классическое решение

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**Introduction.** The Klein–Gordon equation describes the dynamics of quantum particles with zero spin and non-zero mass (e. g., Higgs boson, pion, and kaon) at speeds close to the speed of light [1]. Some other equations from mechanics and electrodynamics can be reduced to the Klein–Gordon equation. Such equations include the telegraph equation, which describes the voltage and current on an electrical transmission line with distance and time [2], and a wave equation with damping term, which describes transverse waves of displacement on a string under consideration of the friction [3].

In previous papers, as a rule, solutions of mixed problems for the Klein–Gordon equation in a quarter-plane were either not written out, or it was written out in an implicit analytic form [4; 5], or written out for a particular case of the equation [6]. We also note that in the papers where the solution is written out, the authors usually do only formal construction of the solution [6; 7], wherein the uniqueness and smoothness of the solution are not studied.

In this article, we will use the method of reflections [8; 9] to solve the boundary value problems associated with the Klein–Gordon equation on the half-line  $0 < x < \infty$  and the finite interval  $0 < x < l$ .

**The first mixed problem on the half-line.** Let us start with the Dirichlet boundary condition first and consider the initial boundary value problem

$$\begin{cases} \partial_t^2 v - a^2 \partial_x^2 v - c^2 v = 0, & 0 < t < \infty, & 0 < x < \infty, \\ v(0, x) = \phi(x), & \partial_t v(0, x) = \psi(x), & x > 0, \\ v(t, 0) = 0, & t > 0. \end{cases} \quad (1)$$

We assume that  $a > 0$ ,  $c \in \{(x + iy) \mid (x = 0 \text{ and } y > 0) \text{ or } (x > 0 \text{ and } y = 0)\}$ . We reduce the Dirichlet problem (1) to the whole line by the reflection method. The idea is again to extend the initial data, in this case  $\phi$ ,  $\psi$ , to the whole line, so that the boundary condition is automatically satisfied for the solutions of the Cauchy problem on the whole line with the extended initial data. Since the boundary condition is in the Dirichlet form, one must take the odd extensions

$$\phi_{\text{odd}}(x) = \begin{cases} \phi(x), & x > 0, \\ 0, & x = 0, \\ -\phi(-x), & x < 0; \end{cases} \quad \psi_{\text{odd}}(x) = \begin{cases} \psi(x), & x > 0, \\ 0, & x = 0, \\ -\psi(-x), & x < 0. \end{cases} \quad (2)$$

Consider the Cauchy problem on the whole line with the extended initial data

$$\begin{cases} \partial_t^2 u - a^2 \partial_x^2 u - c^2 u = 0, & 0 < t < \infty, & -\infty < x < \infty, \\ u(0, x) = \phi_{\text{odd}}(x), & \partial_t u(0, x) = \psi_{\text{odd}}(x), & -\infty < x < \infty. \end{cases} \quad (3)$$

We know the solution to the problem (3) in an exact analytical form [10]

$$\begin{aligned} u(t, x) = & \frac{\phi_{\text{odd}}(x + at) + \phi_{\text{odd}}(x - at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi_{\text{odd}}(\xi) d\xi + \\ & + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi_{\text{odd}}(\xi) d\xi. \end{aligned} \quad (4)$$

It is obvious that  $u(t, 0) = 0$ . Then defining the restriction of  $u(x, t)$  to the positive half-line  $x \geq 0$ ,

$$v(t, x) = u(t, x) \Big|_{x \geq 0} \quad (5)$$

we automatically have that  $v(0, t) = u(0, t) = 0$ . So the boundary condition of the Dirichlet problem (1) is satisfied for  $v$ . The initial conditions are satisfied as well since the restrictions of  $\phi_{\text{odd}}$  and  $\psi_{\text{odd}}$  to the positive half-line are  $\phi$  and  $\psi$  respectively. Finally,  $v$  solves the Klein–Gordon equation for  $x > 0$  since  $u$  satisfies the Klein–Gordon equation for all  $x \in \mathbb{R}$ , and in particular for  $x > 0$ . Thus,  $v$  defined by (5) is a solution to the Dirichlet problem (1). The solution must be unique since the odd extension of the solution will solve the Cauchy problem (3), and therefore must be unique.

Using generalized d'Alembert's formula (4) for the solution of (3), and taking the restriction (5), we have that for  $x \geq 0$ ,

$$\begin{aligned} v(t, x) = & \frac{\phi_{\text{odd}}(x + at) + \phi_{\text{odd}}(x - at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi_{\text{odd}}(\xi) d\xi + \\ & + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi_{\text{odd}}(\xi) d\xi, \quad t > 0, \quad x > 0. \end{aligned} \quad (6)$$

Notice that if  $x \geq 0$  and  $t > 0$ , then  $x + at > 0$ , and  $\phi_{\text{odd}}(x + at) = \phi(x + at)$ . If in addition  $x - at > 0$ , then  $\phi_{\text{odd}}(x - at) = \phi(x - at)$ , and over the interval  $s \in [x - at, x + at]$ ,  $\psi_{\text{odd}}(s) = \psi(s)$ . Thus, for  $x > at$ , we have

$$v(t, x) = \frac{\phi(x + at) + \phi(x - at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi(\xi) d\xi + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi(\xi) d\xi, \quad t > 0, \quad x > 0, \quad x - at > 0. \tag{7}$$

For  $0 < x < at$ , the argument  $x - at < 0$ , and using (2) we can rewrite the solution (6) as

$$v(t, x) = \frac{\phi(x + at) - \phi(at - x)}{2} + \frac{ct}{2} \int_0^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi(\xi) d\xi + \frac{1}{2a} \int_0^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi(\xi) d\xi + \frac{1}{2a} \int_{x-at}^0 -I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi(-\xi) d\xi + \frac{ct}{2} \int_{x-at}^0 \frac{-1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi(-\xi) d\xi, \quad t > 0, \quad x > 0, \quad x - at < 0.$$

Making the change of variables  $\xi \mapsto -\xi$ , we get

$$v(t, x) = \frac{\phi(x + at) - \phi(at - x)}{2} + \frac{ct}{2} \int_0^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi - x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \phi(\xi) d\xi + \frac{1}{2a} \int_0^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi - x)^2} \right) \psi(\xi) d\xi + \frac{1}{2a} \int_{at-x}^0 I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi + x)^2} \right) \psi(\xi) d\xi + \frac{ct}{2} \int_{at-x}^0 \frac{1}{\sqrt{a^2 t^2 - (\xi + x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi + x)^2} \right) \phi(\xi) d\xi, \quad t > 0, \quad x > 0, \quad x - at < 0. \tag{8}$$

**Theorem 1.** Let  $\phi \in C^2[0, \infty)$ ,  $\psi \in C^1[0, \infty)$ . The first mixed problem (1) has a unique classical solution, represented by the formulas (7), (8), of the class  $C^2([0, \infty) \times [0, \infty))$  if and only if  $\phi(0) = 0$ ,  $\psi(0) = 0$ ,  $\phi''(0) = 0$ .

**Proof.** From formulas (7) and (8) we conclude that  $v \in C^2(\{(t, x) | x \geq 0, t \geq 0, x - at \geq 0\})$  and  $v \in C^2(\{(t, x) | x \geq 0, t \geq 0, x - at \leq 0\})$  if  $\phi \in C^2[0, \infty)$ ,  $\psi \in C^1[0, \infty)$ . So, the condition

$$\partial_t^p \partial_x^k v(t, at - 0) = \partial_t^p \partial_x^k v(t, at + 0), \quad 0 \leq p + k \leq 2, \quad p, k \in \{0, 1, 2\}, \tag{9}$$

must be satisfied to  $v \in C^2([0, \infty) \times [0, \infty))$ . This condition is fulfilled if  $\phi(0) = 0$ ,  $\psi(0) = 0$ ,  $\phi''(0) = 0$ . If  $v \in C^2([0, \infty) \times [0, \infty))$  then condition (9) is satisfied. This entails equalities  $\phi(0) = 0$ ,  $\psi(0) = 0$ ,  $\phi''(0) = 0$ .

**The second mixed problem on the half-line.** For the Neumann problem on the half-line,

$$\begin{cases} \partial_t^2 w - a^2 \partial_x^2 w - c^2 w = 0, & 0 < t < \infty, \quad 0 < x < \infty, \\ w(0, x) = \phi(x), \quad \partial_t v(0, x) = \psi(x), & x > 0, \\ \partial_x w(t, 0) = 0, & t > 0. \end{cases} \tag{10}$$

As in the previous problem, we assume that  $a > 0$ ,  $c \in \{(x + iy) | (x = 0 \text{ and } y > 0) \text{ or } (x > 0 \text{ and } y = 0)\}$ . We use the reflection method with even extensions to reduce the problem to the Cauchy problem on the whole line. Define the even extensions of the initial data

$$\phi_{\text{even}}(x) = \begin{cases} \phi(x), & x > 0, \\ \phi(-x), & x < 0; \end{cases} \quad \Psi_{\text{even}}(x) = \begin{cases} \psi(x), & x > 0, \\ \psi(-x), & x < 0, \end{cases} \quad (11)$$

and consider the following Cauchy problem on the whole line

$$\begin{cases} \partial_t^2 u - a^2 \partial_x^2 u - c^2 u = 0, & 0 < t < \infty, \quad 0 < x < \infty, \\ u(0, x) = \phi_{\text{even}}(x), \quad \partial_t u(0, x) = \Psi_{\text{even}}(x), & -\infty < x < \infty. \end{cases} \quad (12)$$

The solution  $u$  to the problem (12) will satisfy  $u'_x(0, t) = 0$  for all  $t > 0$ . Similar to the case of the Dirichlet problem, the restriction  $w(t, x) = u(t, x)|_{x \geq 0}$  will be the unique solution to the Neumann problem (10).

Using generalized d'Alembert's formula for the solution  $u$  of (12), and taking the restriction to  $x \geq 0$ , we obtain

$$\begin{aligned} v(t, x) = & \frac{\phi_{\text{even}}(x+at) + \phi_{\text{even}}(x-at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \Psi_{\text{even}}(\xi) d\xi + \\ & + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi-x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \phi_{\text{even}}(\xi) d\xi, \quad t > 0, \quad x > 0. \end{aligned} \quad (13)$$

Again it is necessary to consider the two cases  $x > at$  and  $0 < x < at$  separately. Using the definitions (11), the solution (13) can be written as

$$\begin{aligned} v(t, x) = & \frac{\phi(x+at) + \phi(x-at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \psi(\xi) d\xi + \\ & + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi-x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \phi(\xi) d\xi, \quad x > 0, \quad t > 0, \quad x-at > 0, \\ v(t, x) = & \frac{\phi(x+at) + \phi(at-x)}{2} + \frac{ct}{2} \int_0^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi-x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \phi(\xi) d\xi + \\ & + \frac{1}{2a} \int_0^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \psi(\xi) d\xi + \frac{1}{2a} \int_0^{at-x} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi+x)^2} \right) \psi(\xi) d\xi + \\ & + \frac{ct}{2} \int_0^{at-x} \frac{1}{\sqrt{a^2 t^2 - (\xi+x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi+x)^2} \right) \phi(\xi) d\xi, \quad x > 0, \quad t > 0, \quad x-at < 0. \end{aligned} \quad (14)$$

**Theorem 2.** Let  $\phi \in C^2[0, \infty)$ ,  $\psi \in C^1[0, \infty)$ . The second mixed problem (10) has the unique classical solution, represented by the formula (14), of the class  $C^2([0, \infty) \times [0, \infty))$  if and only if  $\phi'(0) = 0$ ,  $\psi'(0) = 0$ .

**Proof.** To prove this theorem, it needs to repeat the proof of Theorem 1.

**The first mixed problem on a finite interval.** We would like to apply the same method to the boundary value problems on the finite interval, which corresponds to the physical case of a potential well. Consider the Dirichlet problem on the finite line

$$\begin{cases} \partial_t^2 v - a^2 \partial_x^2 v - c^2 v = 0, & 0 < t < \infty, \quad 0 < x < l, \\ v(0, x) = \phi(x), \quad \partial_t v(0, x) = \psi(x), & x > 0, \\ v(t, 0) = v(t, l) = 0, & t > 0. \end{cases} \quad (15)$$

The homogeneous Dirichlet conditions correspond to the quantum particle in a box model (infinite potential well).

Recall that the idea of the reflection method is to extend the initial data to the whole line in such a way, that the boundary conditions are automatically satisfied. For the homogeneous Dirichlet data the appropriate choice is the odd extension. In this case, we need to extend the initial data  $\phi, \psi$ , which are defined only on the interval  $0 < x < l$ , in such a way that the resulting extensions are odd with respect to both  $x = 0$ , and  $x = l$ . That is, the extensions must satisfy

$$f(-x) = -f(x) \text{ and } f(l-x) = -f(l+x). \tag{16}$$

Notice that for such a function  $f(0) = -f(0)$  from the first condition, and  $f(l) = -f(l)$  from the second condition, hence,  $f(0) = f(l) = 0$ . Subsequently, boundary conditions will be automatically satisfied for the solution to the Cauchy problem with such data. Notice that the conditions (16) imply functions that are odd with respect to both  $x = 0$  and  $x = l$  satisfy  $f(2l+x) = -f(-x) = f(x)$ , which means that such functions must be  $2l$ -periodic. Using this we can define the extensions of the initial data  $\phi, \psi$  as

$$\phi_{\text{ext}}(x) = \begin{cases} \phi(x), & 0 < x < l, \\ -\phi(-x), & -l < x < 0, \\ \text{extended to be } 2l\text{-periodic,} \end{cases} \quad \psi_{\text{ext}}(x) = \begin{cases} \psi(x), & 0 < x < l, \\ -\psi(-x), & -l < x < 0, \\ \text{extended to be } 2l\text{-periodic.} \end{cases}$$

Now, consider the Cauchy problem on the whole line with the extended initial data

$$\begin{cases} \partial_t^2 u - a^2 \partial_x^2 u - c^2 u = 0, & 0 < t < \infty, \quad 0 < x < \infty, \\ u(0, x) = \phi_{\text{ext}}(x), \quad \partial_t u(0, x) = \psi_{\text{ext}}(x), & -\infty < x < \infty. \end{cases} \tag{17}$$

By generalized d'Alembert's formula, the solution of (17) will be given as

$$\begin{aligned} u(t, x) = & \frac{\phi_{\text{ext}}(x+at) + \phi_{\text{ext}}(x-at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} I_0 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \psi_{\text{ext}}(\xi) d\xi + \\ & + \frac{ct}{2} \int_{x-at}^{x+at} \frac{1}{\sqrt{a^2 t^2 - (\xi-x)^2}} I_1 \left( \frac{c}{a} \sqrt{a^2 t^2 - (\xi-x)^2} \right) \phi_{\text{ext}}(\xi) d\xi, \quad t > 0, \quad x > 0. \end{aligned} \tag{18}$$

From (18) we automatically have  $u(0, t) = u(l, t) = 0$ , and the restriction

$$v(t, x) = u(t, x) |_{0 \leq x \leq l} \tag{19}$$

will solve the boundary value problem (15).

**Theorem 3.** *Let  $\phi \in C^2[0, \infty)$ ,  $\psi \in C^1[0, \infty)$ . The first mixed problem (15) has a unique classical solution, represented by the formulas (18), (19), of the class  $C^2([0, \infty) \times [0, l])$  if and only if  $\phi(0) = 0$ ,  $\psi(0) = 0$ ,  $\phi''(0) = 0$ ,  $\phi(l) = 0$ ,  $\psi(l) = 0$ ,  $\phi''(l) = 0$ .*

**Proof.** See [4].

**Conclusion.** We derived the solutions to the Klein–Gordon equation on the half-line in much the same way as was done for the wave equation and the heat equation [8; 9]. That is, we reduced the initial/boundary value problem to the Cauchy problem over the whole line through the appropriate extension of the initial data. The solutions are obtained in an explicit analytical form. The necessary and sufficient conditions for the existence and uniqueness of classical solutions are derived. It is planned to apply the method of reflections to other partial differential equations in further research.

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