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CLASSICAL SOLUTION OF THE INITIAL-VALUE PROBLEM FOR A ONE-DIMENSIONAL QUASILINEAR WAVE EQUATION

Abstract. For a one-dimensional mildly quasilinear wave equation given in the upper half-plane, we consider the Cauchy problem. The solution is constructed by the method of characteristics in an implicit analytical form as a solution of some integro-differential equation. The solvability of this equation, as well the smoothness of its solution, is studied. For the problem in question, the uniqueness of the solution is proved and the conditions under which its classical solution exists are established. When given data is not enough smooth a mild solution is constructed.

Keywords: nonlinear wave equation, Cauchy problem, method of characteristics, fixed-point principle, classical solution.

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КЛАССИЧЕСКОЕ РЕШЕНИЕ ЗАДАЧИ КОШИ ДЛЯ ОДНОМЕРНОГО КВАЗИЛИНЕЙНОГО ВОЛНОВОГО УРАВНЕНИЯ

Аннотация. Для одномерного слабо квазилинейного волнового уравнения, заданного в верхней полуплоскости, рассматривается задача Коши. Решение строится в неявном аналитическом виде как решение некоторого интегро-дифференциального уравнения. Проводится исследование разрешимости этого уравнения, а также гладкости его решения. Для рассматриваемой задачи доказывается единственность решения и устанавливаются условия, при выполнении которых существует ее классическое решение. При недостаточной гладкости начальных данных строится слабое решение.

Ключевые слова: нелинейное волновое уравнение, задача Коши, метод характеристик, принцип неподвижной точки, классическое решение

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Introduction. Continuous media are described mainly by nonlinear partial differential equations. The choice of linear or nonlinear equations for describing a medium depends on the role played by nonlinear effects and is determined by the specific physical situation. For example, when describing the propagation of laser pulses, it is necessary to take into account the dependence of the refractive index of the medium on the electromagnetic field intensity.

The linearization of nonlinear equations of mathematical physics does not always lead to meaningful results. It may turn out that the linearized equations apply to the physical process in question only for some finite time. Moreover, from the viewpoint of physics, it is often "essentially nonlinear" solutions, qualitatively different from the solutions of linear equations, that are extremely important for nonlinear equations of mathematical physics. These can be stationary solutions of the soliton type, localized in one or several dimensions, or solutions of the wave collapse type, which describe the spontaneous concentration of energy in small regions of space [1].

The solvability in some function spaces of the Cauchy problem and boundary value problems is established for a wide class of weakly nonlinear hyperbolic equations of the form [2]

$$(\partial_t^2 - \Delta)u(t, \mathbf{x}) = f(t, \mathbf{x}, u(t, \mathbf{x}), \partial_t u(t, \mathbf{x}), \nabla u(t, \mathbf{x})),\ t > 0,\ \mathbf{x} \in \Omega \subseteq \mathbb{R}^n.$$

We note that various fixed-point theorems and the method of successive approximations are often used to find solutions to nonlinear equations. For example, Banach's fixed point theorem was successfully used to obtain a weak solution to the Cauchy problem for a mildly nonlinear wave equation with a nonlinearity of the form $f(\nabla u, \partial_t u, u)$ [3]. In the paper [4], the method of successive approximations was used to construct a twice continuously differentiable solution of the Cauchy problem on a finite time interval for the nonlinear wave equation with a nonlinearity of the form $G'(|u|)u$ with a certain smoothness and boundedness of the nonlinearity G, initial functions, and their derivatives; moreover, under additional conditions on the nonlinearity, the solution is determined in some cone. In the article [5], an auxiliary system with a viscosity parameter was used to build weak solutions for a quasilinear wave equation. A priori estimates and the method of characteristics were used to construct a strong generalized solution for a wave equation with a dissipative term (a nonlinearity of the form $g(t, x, u)\partial_t u$) [6].

We can see that the Cauchy problem is mostly studied with infinitely differentiable small [7–11] or slowly decaying data [12]. It is mainly due to the methods of study and the function spaces where the solution is sought.

In the present article, we use a fixed point principle to solve the Cauchy problem for a nonlinear inhomogeneous hyperbolic equation of the second order. We also derive conditions under which the solution of the Cauchy problem will be classical. Moreover, we do not assume that the initial data of the problem are infinitely differentiable and/or small but take them sufficiently smooth, namely, from the classes C^2 and C^1. In the future, we will use the obtained results to study initial-boundary value problems.

Statement of the problem. In the domain $(0, \infty) \times \mathbb{R}$ of two independent variables $(t, x) \in (0, \infty) \times \mathbb{R} \subset \mathbb{R}^2$, consider the one-dimensional nonlinear equation

$$\partial_t^2 u(t, x) - a^2 \partial_x^2 u(t, x) + f(t, x, u(t, x), \partial_t u(t, x), \partial_x u(t, x)) = F(t, x),\ (t, x) \in (0, \infty) \times \mathbb{R}, \tag{1}$$

where $a \in (0, \infty)$, F is a function given on the set $[0, \infty) \times \mathbb{R}$, f is a function given on the set $[0, \infty) \times \mathbb{R}^4$. Equation (1) is equipped with the initial condition

$$u(0, x) = \phi(x),\ \ \partial_t u(0, x) = \psi(x),\ x \in \mathbb{R}, \tag{2}$$

where ϕ and ψ are some real-valued functions defined on the real axis.

It should be noted that Equation (1) can be reduced to a first-order semilinear hyperbolic system

$$\begin{cases} \partial_t u(t, x) = q(t, x), \\ \partial_t p(t, x) - \partial_x q(t, x) = 0, \\ \partial_t q(t, x) - a^2 \partial_x p(t, x) = F(t, x) - f(t, x, u(t, x), p(t, x), q(t, x)), \end{cases}$$

with respect to the unknown functions u, $q = \partial_t u$, and $p = \partial_x u$, and weak solutions of the original problem correspond to the so-called solutions in the broad sense for the equivalent system. Results on the existence and uniqueness of such solutions for general semilinear hyperbolic systems are well known and presented in the works [13; 14]. However, in these papers, generally speaking, only local solutions are established. They will be global classical provided they are bounded in any characteristic triangle and the given functions f, F, ϕ, ϕ', and ψ are continuously differentiable. And the answer to the question "What conditions must be imposed on these functions to obtain global classical solutions?" is not available in [13; 14].

Some explicit conditions for the existence of a local classical solution to the problem (1)–(2) are given in the book [15] and article [16].

In contrast to many works devoted to the Cauchy problem, we will not assume the initial data of the problem to be infinitely differentiable and/or small but take them sufficiently smooth, namely, $F \in C^1([0, \infty) \times \mathbb{R})$, $f \in C^1([0, \infty) \times \mathbb{R}^4)$, $\phi \in C^2(\mathbb{R})$, and $\psi \in C^1(\mathbb{R})$.

Integro-differential equation. Introduce into consideration the operator K acting by the formula

$$K[u](t,x)=\frac{\phi(x-at)+\phi(x+at)}{2}+\frac{1}{2a}\int_{x-at}^{x+at}\psi(\xi)d\xi+$$
$$+\frac{1}{2a}\int_0^t d\tau\int_{x-a(t-\tau)}^{x+a(t-\tau)}(F(\tau,\xi)-f(\tau,\xi,u(\tau,\xi),\partial_t u(\tau,\xi),\partial_x u(\tau,\xi)))d\xi,\ (t,x)\in[0,\infty)\times\mathbb{R}. \quad (3)$$

In the closure $[0,\infty)\times\mathbb{R}$ of the domain $(0,\infty)\times\mathbb{R}$, we consider the nonlinear integro-differential equation

$$u(t,x)=K[u](t,x),\ (t,x)\in[0,\infty)\times\mathbb{R}. \quad (4)$$

L e m m a 1. *Let the conditions* $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^2(\mathbb{R})$, *and* $\psi\in C^1(\mathbb{R})$ *be satisfied. The function u is a continuous-differentiable solution of Equation (4) if it is a classical solution of the initial-value problem (1), (2).*

P r o o f. See [17].

Lemma 1 can also be proved using the method of characteristics [18] or Green's theorem.

L e m m a 2. *Let the conditions* $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^2(\mathbb{R})$, *and* $\psi\in C^1(\mathbb{R})$ *be satisfied. The function u belongs to the class* $C^2([0,\infty)\times\mathbb{R})$ *and satisfies Equation (1) and conditions (2) if it is a continuous-differentiable solution of Equation (4).*

P r o o f. If the function u is a continuous-differentiable solution of Equation (4), then, by virtue of the smoothness conditions $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^2(\mathbb{R})$, and $\psi\in C^1(\mathbb{R})$, similarly to [19; 20], we conclude that $u\in C^2([0,\infty)\times\mathbb{R})$. Substituting the representations (4) into (1) and (2), we verify that the function u satisfies Equation (1) in $(0,\infty)\times\mathbb{R}$ and conditions (2) in $\mathbb{R}$. The proof of the lemma is complete.

T h e o r e m 1. *Let the conditions* $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^2(\mathbb{R})$ *and* $\psi\in C^1(\mathbb{R})$ *be satisfied. The function u belongs to the class* $C^2([0,\infty)\times\mathbb{R})$ *and satisfies Equation (1) and conditions (2) if and only if it is a continuous-differentiable solution of Equation (4).*

The p r o o f of the theorem follows from Lemmas 1 and 2.

For definiteness, we define the topology of the Fréchet space $C^j([0,T]\times\mathbb{R})$ by a countable family of seminorms $\mathfrak{p}_m=\|\bullet\|_{C^j(\Omega_m)}$, $m\in\mathbb{N}\cap[\text{ceil}(aT+1),\infty)$, where

$$\Omega_m=\text{Conv}\{(0,-m),(0,m),(T,aT-m),(T,aT+m)\}.$$

T h e o r e m 2. *Let the conditions* $F\in C([0,\infty)\times\mathbb{R})$, $f\in C([0,\infty)\times\mathbb{R}^4))$, $\phi\in C^1(\mathbb{R})$ *and* $\psi\in C(\mathbb{R})$ *be satisfied, and let the function f satisfy the Lipschitz condition with constant L with respect to the three last variables, i. e.,* $|f(t,x,z_1,z_2,z_3)-f(t,x,w_1,w_2,w_3)|\le L(|z_1-w_1|+|z_2-w_2|+|z_3-w_3|)$. *Then the operator* $K:C^1(\Omega_m)\mapsto C^1(\Omega_m)$, *acting by the formula (3), is* $\mathfrak{L}$ *– Lipschitz, where* $\mathfrak{L}=3L\max\{T,T^2\}\times\max\{1,a^{-1}\}$.

P r o o f. Direct verification.

C o r o l l a r y 1. Let the conditions $F\in C([0,\infty)\times\mathbb{R})$, $f\in C([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^1(\mathbb{R})$ and $\psi\in C(\mathbb{R})$ be satisfied, let the function f satisfy the Lipschitz condition with constant L with respect to the three last variables, and let $T<\min\{1,(3L\max\{1,a^{-1}\})^{-1}\}$. Then the operator $K:C^1([0,T]\times\mathbb{R})\mapsto C^1([0,T]\times\mathbb{R})$, acting by the formula (3), is $\mathfrak{p}_m$-contraction for any $m\in\mathbb{N}\cap[\text{ceil}(aT+1),\infty)$.

Theorem 2.2 from [21] now implies the existence of a unique fixed point, which is the unique solution of (4).

C o r o l l a r y 2. Let the conditions $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^1(\mathbb{R})$ and $\psi\in C(\mathbb{R})$ be satisfied, let the function f satisfy the Lipschitz condition with constant L with respect to the three last variables, and let $T<\min\{1,(3L\max\{1,a^{-1}\})^{-1}\}$. Then there exists a unique solution of Equation (4) in the class $C^1([0,T]\times\mathbb{R}))$.

Classical solution. We have therefore built a unique classical solution $u^{(0)}$ of (1), (2) on $[0,T]\times\mathbb{R}$ provided $F\in C^1([0,\infty)\times\mathbb{R})$, $f\in C^1([0,\infty)\times\mathbb{R}^4)$, $\phi\in C^2(\mathbb{R})$, $\psi\in C^1(\mathbb{R})$, f is Lipschitz continuous with constant L with respect to the three last variables, and $T=\min\{1,(3L\max\{1,a^{-1}\})^{-1}\}/2.$ Now we can extend the solution to the time intervals $[nT,(n+1)T],\ n\in\mathbb{N},$ using matching conditions

$$u^{(n)}(nT,x)=u^{(n-1)}(nT,x),\ \partial_t u^{(n)}(nT,x)=\partial_t u^{(n-1)}(nT,x),\ x\in\mathbb{R}. \tag{5}$$

Differentiating equalities (5) with respect to x, we obtain

$$\begin{gathered}\partial_x u^{(n)}(nT,x)=\partial_x u^{(n-1)}(nT,x),\ \partial_x^2 u^{(n)}(nT,x)=\partial_x^2 u^{(n-1)}(nT,x),\\ \partial_x\partial_t u^{(n)}(nT,x)=\partial_x\partial_t u^{(n-1)}(nT,x),\ x\in\mathbb{R}.\end{gathered} \tag{6}$$

We express the quantities $\partial_t^2 u^{(j)}(nT,x),\ \ j\in\{n-1,n\}$ from Equation (1)

$$\begin{gathered}\partial_t^2 u^{(j)}(nT,x)=F(nT,x)-a^2\partial_x^2 u^{(j)}(nT,x)+\\ +f(t,x,u^{(j)}(nT,x),\partial_t u^{(j)}(nT,x),\partial_x u^{(j)}(nT,x)),\ x\in\mathbb{R}.\end{gathered} \tag{7}$$

By virtue of (5) and (6) and the continuity of the functions f and F in expression (7), the right-hand sides are equal for $j=n-1$ and $j=n,$ then the left-hand sides are also equal. Conditions (5)–(7) mean that the function

$$u^{(n-1,n)}(t,x)=\begin{cases}u^{(n-1)}(t,x), & (t,x)\in[(n-1)T,nT]\times\mathbb{R},\\ u^{(n)}(t,x), & (t,x)\in[nT,(n+1)T]\times\mathbb{R},\end{cases}$$

belongs to the class $C^2([(n-1)T,(n+1)T]\times\mathbb{R})$ and satisfies Equation (1) on the set $[(n-1)T,(n+1)T]\times\mathbb{R}.$ We note that another choice of matching conditions (5) will cause at least one of the functions $u^{(n-1,n)}$ or $\partial_t u^{(n-1,n)}$ to be discontinuous, which will entail $u^{(n-1,n)}\notin C^2([(n-1)T,(n+1)T]\times\mathbb{R}).$

Similarly, we conclude that the function

$$u^{(\infty)}(t,x)=u^{(n)}(t,x),\ (t,x)\in[nT,(n+1)T]\times\mathbb{R},$$

belongs to the class $C^2([0,\infty)\times\mathbb{R})$ and satisfies Equation (1) on the set $[0,\infty)\times\mathbb{R}$ and the Cauchy conditions (2) by construction. We state the result as the following assertion.

T h e o r e m 3. *Let the conditions* $F\in C^1([0,\infty)\times\mathbb{R}),\ \ f\in C^1([0,\infty)\times\mathbb{R}^4),\ \ \phi\in C^2(\mathbb{R}),$ *and* $\psi\in C^1(\mathbb{R})$ *be satisfied, and let the function f satisfy the Lipschitz condition with constant L with respect to the three last variables. Then the Cauchy problem (1), (2) has a unique solution in the class* $C^2([0,\infty)\times\mathbb{R}).$

Mild solution. If the given functions of the problem (1), (2) do not satisfy the smoothness conditions specified in Theorem 3, then we can speak of mild, weak, and generalized solutions instead of the classical ones.

D e f i n i t i o n 1. A function $u\in C^1([0,\infty)\times\mathbb{R})$ is called a mild solution of the problem (1), (2) if it satisfies Equation (4).

R e m a r k 1. Obviously, any classical solution of the problem (1), (2) is a mild solution of this problem too. In its turn, if a mild solution of problem (1), (2) belongs to the class $C^2([0,\infty)\times\mathbb{R}),$ then it will be a classical solution of that problem.

We obtain the following result by repeating the arguments of the previous section.

T h e o r e m 4. *Let the conditions* $F\in C([0,\infty)\times\mathbb{R}),\ \ f\in C([0,\infty)\times\mathbb{R}^4),\ \ \phi\in C^1(\mathbb{R})$ *and* $\psi\in C(\mathbb{R})$ *be satisfied, and let the function f satisfy the Lipschitz condition with constant L with respect to the three last variables. Then the Cauchy problem (1), (2) has a unique mild solution.*

Conclusions. In the present paper, we obtain sufficient conditions under which there exist a unique classical solution and a unique mild solution of the Cauchy problem in a half-plane for a mildly quasilinear wave equation. The dependence of the smoothness of the solution on the smoothness of the initial functions is established.

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