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Inna N. Safonova[1], Alexander N. Skiba[2]

[1]Belarusian State University, Minsk, Republic of Belarus
[2]Francisk Skorina Gomel State University, Gomel, Republic of Belarus

ON SOME CLASSES OF FINITE σ-SOLUBLE *PσT*-GROUPS

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Abstract. Let $\mathfrak{X}$ be a class of groups. Suppose that with each group $G \in \mathfrak{X}$ we associate some system of its subgroups $\tau(G)$. Then τ is said to be a *subgroup functor* on $\mathfrak{X}$ if the following conditions are hold: (1) $G \in \tau(G)$ for each group $G \in \mathfrak{X}$; (2) for any epimorphism $\varphi : A \to B$, where $A, B \in \mathfrak{X}$, and for any groups $H \in \tau(A)$ and $T \in \tau(B)$ we have $H^{\varphi} \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$. In this paper, were considered some applications of such subgroup functors in the theory of finite groups in which generalized normality for subgroups is transitive.

Keywords: finite group, modular subgroup, σ-subnormal subgroup, σ-soluble group, subgroup functor

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И. Н. Сафонова[1], А. Н. Скиба[2]

[1]Белорусский государственный университет, Минск, Республика Беларусь
[2]Гомельский государственный университет им. Франциска Скорины, Гомель, Республика Беларусь

О НЕКОТОРЫХ КЛАССАХ КОНЕЧНЫХ σ-РАЗРЕШИМЫХ *PσT*-ГРУПП

(Представлено академиком В. И. Янчевским)

Аннотация. Пусть $\mathfrak{X}$ – класс групп. Предположим, что каждой группе $G \in \mathfrak{X}$ сопоставлена некоторая система ее подгрупп $\tau(G)$. Тогда говорят, что τ – подгрупповой функтор на $\mathfrak{X}$, если выполняются следующие условия: (1) $G \in \tau(G)$ для каждой группы $G \in \mathfrak{X}$; (2) для любого эпиморфизма $\varphi : A \to B$, где $A, B \in \mathfrak{X}$, и для любых групп $H \in \tau(A)$ и $T \in \tau(B)$ имеем $H^{\varphi} \in \tau(B)$ и $T^{\varphi^{-1}} \in \tau(A)$. Рассмотрены некоторые приложения таких подгрупповых функторов в теории конечных групп, у которых транзитивна обобщенная нормальность для подгрупп.

Ключевые слова: конечная группа, модулярная подгруппа, σ-субнормальная подгруппа, σ-разрешимая группа, подгрупповой функтор

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Introduction. Throughout this paper, all groups are finite and G always denotes a finite group; $\mathcal{L}(G)$ is the lattice of all subgroups of G. Moreover, $\sigma = \{\sigma_i \mid i \in I\}$ is some partition of the set of all primes $\mathbb{P}$ and if G is a σ_i-group for some i, then G is called σ-primary [1]; G is said to be [2]: σ-soluble if every chief factor of G is σ-primary; σ-nilpotent if G is the direct product of σ-primary groups.

If n is an integer, the symbol $\pi(n)$ denotes the set of all primes dividing n; as usual, $\pi(G) = \pi(|G|)$, the set of all primes dividing the order of G; $\sigma(n) = \{\sigma_i \mid \sigma_i \cap \pi(n) \neq \varnothing\}$ and $\sigma(G) = \sigma(|G|)$ [2].

Let $\mathfrak{X}$ be a class of groups. Suppose that with each group $G \in \mathfrak{X}$ we associate some system of its subgroups $\tau(G)$. Then we say that τ is a *subgroup functor* (in the sense of Skiba [3]) on $\mathfrak{X}$ if the following conditions hold: (1) $G \in \tau(G)$ for each group $G \in \mathfrak{X}$; (2) for any epimorphism $\varphi : A \to B$, where $A, B \in \mathfrak{X}$, and for any groups $H \in \tau(A)$ and $T \in \tau(B)$ we have $H^{\varphi} \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$.

The subgroup functors of this kind have found numerous applications in the formation theory and in the Schunk classes theory (see, for example, the books [3–6]).

In this paper, we discuss some applications of the subgroup functors of this kind in the theory of generalized *T*-groups.

A subgroup *A* of *G* is said to be: *quasinormal* or *permutable* in *G* if *A* permutes with every subgroup *H* of *G*, that is, *AH* = *HA*; *Sylow permutable* or *S-permutable* if *A* permutes with all Sylow subgroups of *G*.

A group *G* is said to be a *T-group* if normality is a transitive relaion on *G*, that is, if *H* is a normal subgroup of *K* and *K* is a normal subgroup of *G*, then *H* is a normal subgroup of *G*. In other words, the group *G* is a *T*-group if and only if every subnormal subgroup of *G* is normal. The description of *T*-groups was first obtained by Gaschütz [7] for the soluble case, and by Robinson in [8], for the general case. The works [7; 8] aroused great interest in the further study of *T*-groups and generalized *T*-groups (*PT*-groups, i. e. groups in which quasinormality is transitive; *PST*-groups, i. e. groups, in which Sylow permutability is transitive, *MT*-groups, i. e. groups, in which modularity is transitive and etc.).

In the last 10 years, considerable attention has been paid to the study of generalized *T*-groups in the theory of σ-properties of a group. Recall that a σ-*property* of a group is understood to be any of its properties that depends on σ and which does not imply any restrictions on σ.

A subgroup *A* of *G* is said to be: (i) σ-*subnormal* in *G* if there is a subgroup chain $A = A_0 \leq A_1 \leq \cdots \leq A_n = G$ such that either $A_{i-1} \trianglelefteq A_i$ or $A_i / (A_{i-1})_{A_i}$ is σ-primary for all $i = 1, \ldots, n$; (ii) σ-*seminormal in G* (J. C. Beidleman) if $x \in N_G(A)$ for all $x \in G$ such that $\sigma(|x|) \cap \sigma(A) = \varnothing$; (iii) σ-*permutable* in *G* if *G* is σ-*full*, that is, *G* has a Hall σ_i-subgroup for every $\sigma_i \in \sigma(G)$ and *A* permutes with all such Hall subgroups of *G*.

In fact, the appearance of the theory of σ-properties of a group was mainly connected with attempts to solve the following difficult problem.

Q u e s t i o n (See [1]). *What is the structure of a P*σ*T-group, that is, a* σ*-full group G in which* σ*-permutability is transitive on G, that is, if H is a* σ*-permutable subgroup of K and K is a* σ*-permutable subgroup of G, then H is a* σ*-permutable subgroup of G?*

This problem turned out to be difficult even in the σ-soluble case: its solution in this case required the development of many aspects of the theory of σ-properties of a group. The theory of σ-soluble *P*σ*T*-groups was mainly developed in the papers of J. Beidleman, A. Ballester-Bolinches, I. N. Safonova, A. N. Skiba, M. K. Pedraza-Aguilera, W. Pérez-Calabuing, Ch. Zhang, W. Guo, A-Ming Liu, and a number of other authors, and the following theorem (which, in fact, is the main result of papers [1; 9]) is the key result in this direction.

T h e o r e m 1 [9]. *If G is a* σ*-soluble P*σ*T-group and* $D = G^{\mathfrak{N}_\sigma}$, *then the following conditions hold:* (i) $G = D \rtimes M$, *where D is an abelian Hall subgroup of G of odd order, M is* σ*-nilpotent and every element of G induces a power automorphism in D;* (ii) $O_{\sigma_i}(D)$ *has a normal complement in a Hall* σ_i*-subgroup of G for all i. Conversely, if Conditions* (i) *and* (ii) *hold for some subgroups D and M of G, then G is a* σ*-soluble P*σ*T-group.*

In this theorem, $G^{\mathfrak{N}_\sigma}$ is the σ-*nilpotent residual* of *G*, that is, the intersection of all normal subgroups *N* of *G* with σ-nilpotent quotient *G* / *N*.

In this paper, we show that Theorem 1 can be substantially strengthened on the basis of the theory of subgroup functors. First of all, we say that a group $G \in \mathfrak{X}$ is τ-*saturated* if for every its subgroup *A* we have $A \in \tau(G)$; if $A \in \tau(G)$, then we say that *A* is a τ-*subgroup* of *G*. In the case when $\mathfrak{X}$ is the class of all groups, we will simply say "subgroup functor" instead of "subgroup functor on $\mathfrak{X}$".

D e f i n i t i o n 1. Let $\mathfrak{X}$ be the class of all σ-soluble groups. Then we say that a subgroup functor τ on $\mathfrak{X}$ is σ-*special* if for every group $G \in \mathfrak{X}$ the following three conditions hold:

(*) each of the σ-subnormal τ-subgroups of *G* is σ-permutable in *G*;

(**) $\langle A, B \rangle \in \tau(G)$ for any two σ-subnormal τ-subgroups *A*, *B* of *G*;

(***) if $G = D \rtimes M$ is a *P*σ*T*-group, where $D = G^{\mathfrak{N}_\sigma}$, and *A* is a σ-primary σ-subnormal subgroup of *G* such that $A \in \tau(M)$, then $A \in \tau(G)$.

D e f i n i t i o n 2. Let $\mathfrak{X}$ be the class of all σ-soluble groups. Then we say that a subgroup functor τ on $\mathfrak{X}$ is *closed* if for every group $G \in \mathfrak{X}$ the following three conditions hold:

(I) if $A \leq E \leq G$ and $A \in \tau(G)$, then $A \in \tau(E)$;

(II) if $G = D \times L$, where *D* is a Hall subgroup of *G* and $A \in \tau(D)$, then $A \in \tau(G)$;

(III) $A \in \tau(G)$ for every normal subgroup *A* of *G*.

The meaning of the concepts introduced above is connected, first of all, with the following two theorems.

T h e o r e m 2 [10]. *Suppose that G is a σ-soluble group with* $D = G^{\mathfrak{N}_\sigma}$, *and let τ be a σ-special subgroup fuctor on the class of all σ-soluble groups* $\mathfrak{X}$. *If every σ-subnormal subgroup of G is a τ-subgroup of G, then G is a PσT-group and the following conditions hold:* (i) $G = D \rtimes M$, *where D is an abelian Hall subgroup of G of odd order, M is a σ-nilpotent τ-saturated group and every element of G induces a power automorphism in D*; (ii) $O_{\sigma_i}(D)$ *has a normal complement in a Hall* σ_i*-subgroup of G for all i. Conversely, if Conditions* (i) *and* (ii) *hold for some subgroups D and M of G*, *then every σ-subnormal subgroup of G belongs to* $\tau(G)$.

T h e o r e m 3 [10]. *Let G be a σ-soluble group and let τ be a σ-special closed subgroup fuctor on the class of all σ-soluble groups* $\mathfrak{X}$. *Then every σ-subnormal subgroup of G is a τ-subgroup of G if and only if G is a PσT-group and every Hall* σ_i*-subgroup of G is τ-saturated for all* $i \in I$.

Applications of Theorems 2 and 3 are based on the following our results.

T h e o r e m 4. *Let* $\mathfrak{X}$ *be the class of all σ-soluble groups and τ(X) be the set of all σ-permutable subgroups of X for each group* $X \in \mathfrak{X}$. *Then τ is a subgroup functor on* $\mathfrak{X}$ *and such a functor τ is both σ-special and closed.*

T h e o r e m 5. *Let* $\mathfrak{X}$ *be the class of all σ-soluble groups and τ(X) be the set of all modular (respectively, normal) subgroups of X for each group* $X \in \mathfrak{X}$. *Then τ is a subgroup functor on* $\mathfrak{X}$ *and such a functor τ is both σ-special and closed.*

T h e o r e m 6. *Let* $\mathfrak{X}$ *be the class of all σ-soluble groups and τ(X) be the set of all σ-hypercentrally embedded subgroups of X for each group* $X \in \mathfrak{X}$. *Then τ is a subgroup functor on* $\mathfrak{X}$ *and such a functor τ is both σ-special and closed.*

P r o o f s of Theorems 4, 5 and 6. Using simple inductive reasoning, we can prove the following lemmas.

L e m m a 1. *Let* $N \leq A, B$ *be subgroups of a σ-soluble group G, where N is normal in G. Suppose that* $\{\sigma_1, \ldots, \sigma_t\} = \sigma(G)$ *and* H_i *is a Hall* σ_i*-subgroup of G for all* $i = 1, \ldots, t$.

(1) *If* $AH_i^x = H_i^x A$ *for all* $i = 1, \ldots, t$ *and all* $x \in G$, *then A is σ-permutable in G.*

(2) *A / N is σ-permutable in G / N if and only if A is σ-permutable in G.*

(3) *If A is σ-permutable in G and* $A \leq B$, *then A is σ-permutable in B.*

L e m m a 2. *Suppose that* $G = D \rtimes M$ *is a σ-soluble PσT-group, where* $D = G^{\mathfrak{N}_\sigma}$, *is a Hall abelian subgroup of G. If A is a σ-primary σ-subnormal subgroup of G and* $A \leq M$, *then* $D \leq C_G(A)$.

L e m m a 3. *Let A be a σ-hypercentrally embedded subgroup of G. If G has a Hall* σ_i*-subgroup H for some i, then AH = HA.*

L e m m a 4. *The set of all σ-hypercentrally embedded subgroups of G forms a sublattice of the lattice* $\mathcal{L}(G)$.

L e m m a 5. *Let A, B and H be subgroups of G. If AH = HA and BH = HB, then* $\langle A,B\rangle H = H\langle A,B\rangle$.

P r o o f of Theorem 4. In view of Lemma 1 (2), τ is a subgroup functor (in the above sence) on $\mathfrak{X}$. Let $G \in \mathfrak{X}$. Clearly, Conditions (*) and (***) hold for *G*. Moreover, in view of Lemma 5, Condition (**) also holds for *G*. Hence the functor τ is σ-special.

Now we show that the functor τ is closed. Condition (I) holds for *G* by Lemma 1 (3) . Condition (III) also holds for *G* since every σ-soluble group is σ-full by [2, Theorem B].

Now let $G = D \times M$, where *D* is a Hall subgroup of *G*, and let *L* be a σ-permutable subgroup of *D*. Let *H* be a Hall σ_i-subgroup of *G* for some $i \in I$. Then $H = (H \cap D) \times (H \cap M)$, where $H \cap D$ is a Hall σ_i-subgroup of *D*. Therefore $L(H \cap D) = (H \cap D)L$ and also we have $[L, M] = 1$. Hence $LH = L(H \cap D)(H \cap M) = (H \cap D)(H \cap M)L$. Therefore *L* is σ-permutable in *G*, so Condition (II) holds for *G*. Hence the functor τ is closed. The theorem is proved.

Recall that a subgroup *M* of *G* is said to be *modular* in *G* if *M* is a modular element (in the sense of Kurosh [11, p. 43]) of the lattice $\mathcal{L}(G)$, that is, (i) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and (ii) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$.

P r o o f of Theorem 5. In view of [11, p. 201, Properties (3), (4)], τ is a subgroup functor (in the above sence) on $\mathfrak{X}$.

We show that the functor τ is σ-special. Let $G \in \mathfrak{X}$. If A is a σ-subnormal modular subgroup of G, then A is σ-permutable in G by [12, Theorem C], so Condition (*) holds for G. Condition (**) holds for G by [11, p. 201, Property (5)]. Finally, suppose that $G = D \rtimes M$ is a $P\sigma T$-group, where $D = G^{\mathfrak{N}_\sigma}$, and let A be a σ-primary σ-subnormal subgroup of G such that A is a modular subgroup of M. We show that in this case A is a modular subgroup of G. In view of [11, Lemma 5.1.13], it is enough to show that A is modular in $\langle x, A\rangle$ for any element x of G of prime power order p^n.

If $x \in D$, it is true by Lemma 2. Now assume that $x \notin D$ and so $x \in M^d$ for some $d \in D$ since M is a Hall subgroup of G. But A is modular in M and so A is modular in M^d since $A^d = A$ by Lemma 2. Therefore A is modular in $\langle x, A\rangle$. Hence Condition (***) holds for G, so τ is a σ-special subgroup functor on $\mathfrak{X}$.

Now we show that the functor τ is closed. Indeed, Conditions (I) and (III), clearly, hold for G, and if $G = D \times M$, where D is a Hall subgroup of G and $L \in \tau(D)$, then, arguing as above, we can show that L is a modular subgroup of G and hence Condition (II) also holds for G.

Finally, if $\tau(X)$ is the set of all normal subgroups of X for each group $X \in \mathfrak{X}$, then, arguin as above, we can show that τ is a subgroup functor on $\mathfrak{X}$ and such a functor τ is both σ-special and closed. The theorem is proved.

We say, following [9], that a subgroup A of G is σ-*hypercentrally embedded in G* if either $A \trianglelefteq G$ or every chief factor H / K of G between A_G and A^G is σ-*central* in G [1], that is, $(H / K) \rtimes (G / C_G(H / K))$ is σ-primary.

P r o o f of Theorem 6. It is not difficult to show that τ is a subgroup functor (in the above sence) on $\mathfrak{X}$.

Now we show that τ is a σ-special subgroup functor on $\mathfrak{X}$. Let $G \in \mathfrak{X}$. In view of Lemma 3, Condition (*) holds for G. On the other hand, since the set of all σ-hypercentrally embedded subgroups of G forms a sublattice of the lattice $\mathcal{L}(G)$ by Lemma 4, Condition (**) also holds for G.

Finally, let $G = D \rtimes M$ be a $P\sigma T$-group, where $D = G^{\mathfrak{N}_\sigma}$, and A is a σ-subnormal σ_i-subgroup of G, $i \in I$, such that A is a σ-hypercentrally embedded subgroup of M. Then $D \le C_G(A)$ by Lemma 2, so $A_G = A_{DM} = A_M$ and $A^G = A^{DM} = A^M \le O_{\sigma_i}(G) \cap M$. It follows that A is normal in G in the case when A is normal in M. Moreover, if H / K is a chief factor of G between $A_G = A_M$ and $A^G = A^M$, then H / K is a chief factor of M and $C_G(H / K) = DC_M(H / K)$ since $[D, A^G] = 1$ by Lemma 2, so

$$G / C_G(H / K) = DM / DC_M(H / K) \simeq M / (M \cap DC_M(H / K)) =$$
$$= M / C_M(H / K)(M \cap D) = M / C_M(H / K)$$

is a σ_i-group. Hence H / K is σ-central in G, so A is σ-hypercentrally embedded in G. Therefore Condition (***) holds for G, so τ is a σ-special subgroup functor on $\mathfrak{X}$.

Similarly, it can be proved that τ is a closed subgroup functor on $\mathfrak{X}$. The theorem is proved.

Some applications. Theorems 4, 5 and 6 allow us to generalize many well-known results. In this section, we present some of these results. First of all note that from Theorems 2 and 4 we get in the case $\sigma = \{\{2\}, \{3\}, \{5\}, \ldots\}$ the follwing classical result.

C o r o l l a r y 1 [7]. *A group G is a soluble T-group if and only if the following conditions are satisfied: (i) the nilpotent residual D of G is an abelian Hall subgroup of odd order; (ii) G acts by conjugation on D as a group power automorphisms; (iii) G / D is a Dedekind group.*

We say that G is a T_σ-*group* if every σ-subnormal subgroup of G is normal.

From Theorems 2 and 5 we get the follwing known results.

C o r o l l a r y 2 [13]. *A σ-soluble group G with $D = G^{\mathfrak{N}_\sigma}$ is a T_σ-group if and only if the following conditions hold: (i) $G = D \rtimes M$, where D is an abelian Hall subgroup of G of odd order, and M is a Dedekind group; (ii) every element of G induces a power automorphism in D; (iii) $O_{\sigma_i}(D)$ has a normal complement in a Hall σ_i-subgroup of G for all i.*

Recall that an *Iwasawa group* is a group in which every subgroup is quasinormal.

C o r o l l a r y 3 [14]. *A group G is a soluble PT-group if and only if the following conditions are satisfied: (i) the nilpotent residual D of G is an abelian Hall subgroup of odd order; (ii) G acts by conjugation on D as a group power automorphisms; (iii) G / D is an Iwasawa group.*

C o r o l l a r y 4 [12]. *Let* $D = G^{\mathfrak{N}_\sigma}$. *Then G is a σ-soluble in which every σ-subnormal subgroup is modular if and only if the following conditions hold: (i)* $G = D \rtimes M$, *where D is an abelian Hall subgroup of G of odd order, L is σ-nilpotent and the lattice* $\mathcal{L}(M)$ *is modular; (ii) every element of G induces a power automorphism in D; (iii)* $O_{\sigma_i}(D)$ *has a normal complement in a Hall* σ_i*-subgroup of G for all i.*

C o r o l l a r y 5 [9]. *A group G is a σ-soluble PσT-group if and only if every σ-subnormal subgroup of G is σ-hypercentrally embedded in G.*

From Theorems 3 and 5 we get the follwing known results.

C o r o l l a r y 6 [15]. *A σ-soluble group G is a* T_σ*-group if and only if G is a soluble T-group and the Hall* σ_i*-subgroups of G are Dedekind for all* $i \in I$.

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Information about the authors

Safonova Inna N. – Ph. D. (Physics and Mathematics), Associate Professor. Belarusian State University (4, Nezavisimosti Ave., 220030, Minsk, Republic of Belarus). E-mail: in.safonova@mail.ru. ORCID: 0000-0001-6896-7208.

Skiba Alexander N. – D. Sc. (Physics and Mathematics), Professor. Francisk Skorina Gomel State University (104, Sovetskaya Str., 246019, Gomel, Republic of Belarus). E-mail: alexander.skiba49@gmail.com. ORCID: 0000-0002-6521-2712.

Информация об авторах

Сафонова Инна Николаевна – канд. физ.-мат. наук, доцент. Белорусский государственный университет (пр. Независимости, 4, 220030, Минск, Республика Беларусь). E-mail: in.safonova@mail.ru. ORCID: 0000-0001-6896-7208.

Скиба Александр Николаевич – д-р физ.-мат. наук, профессор. Гомельский государственный университет им. Франциска Скорины (ул. Советская, 104, 246019, Гомель, Республика Беларусь). E-mail: alexander.skiba49@gmail.com. ORCID: 0000-0002-6521-2712.