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CLASSICAL SOLUTION OF THE CAUCHY PROBLEM FOR A SEMILINEAR WAVE EQUATION WITH A DIRAC POTENTIAL

Abstract. For a one-dimensional semilinear wave equation with a free term that is a solution value at one given point (a Dirac potential), we consider the Cauchy problem in the upper half-plane. We construct the solution using the method of characteristics in implicit analytical form as a solution of some integral equations. The solvability of these equations, as well the smoothness of their solutions, is studied. For the problem in question, we prove the uniqueness of the solution, and establish the conditions under which its classical solution exists.

Keywords: nonlinear wave equation, Cauchy problem, method of characteristics, classical solution, loaded summands, Dirac potential

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КЛАССИЧЕСКОЕ РЕШЕНИЕ ЗАДАЧИ КОШИ ДЛЯ ПОЛУЛИНЕЙНОГО ВОЛНОВОГО УРАВНЕНИЯ С ПОТЕНЦИАЛОМ ДИРАКА

Аннотация. Для одномерного полулинейного волнового уравнения со свободным членом, являющимся значением решения в одной заданной точке (потенциал Дирака), рассматривается задача Коши в верхней полуплоскости. Решение строится методом характеристик в неявном аналитическом виде как решение некоторых интегральных уравнений. Проводится исследование разрешимости этих уравнений, а также зависимости от начальных данных и гладкости их решений. Для рассматриваемой задачи доказывается единственность решения и устанавливаются условия, при выполнении которых существует ее классическое решение.

Ключевые слова: нелинейное волновое уравнение, задача Коши, метод характеристик, классическое решение, нагруженные слагаемые

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Statement of the problem. In the domain $Q = (0, \infty) \times \mathbb{R}$ of two independent variables $(t, x) \in \overline{Q} \subset \mathbb{R}^2$, for the nonlinear wave equation of the form

$$\Box u(t, x) - \Theta(t, x)\delta_{(t_0, x_0)}[u](t, x) = f(t, x, u(t, x)), \ \ (t, x) \in Q, \tag{1}$$

we consider the Cauchy problem with the initial conditions

$$u(0, x) = \varphi(x), \ \partial_t u(0, x) = \psi(x), \ x \in \mathbb{R}, \tag{2}$$

where $\square = \partial_t^2 - a^2\partial_x^2$ is the d'Alembert operator ($a > 0$ for definiteness); (t_0, x_0) is a point from the set Q; $\delta_{(t_0, x_0)}$ is the Dirac delta distribution concentrated at the point (t_0, x_0) (i. e., $\delta_{(t_0, x_0)}[u](t, x) = u(t_0, x_0)$); Θ is a function given on the set $\overline{Q}$; f is a function given on the set $\overline{Q} \times \mathbb{R}$, and φ and ψ are some real-valued functions defined on the real axis.

Equations of the form (1) with an ordinary differential operator instead of the d'Alembert operator were called as loaded in the paper [1]. In the articles [2; 3], equations of the form (1) were called "equations with loaded summands". Following [4], to refine the rather general concepts of "loaded equations" and "equations with loaded summands", we will use the specific term "equation with a Dirac potential" for Eq. (1).

Previously, in the case of an unloaded equation, i. e., $\Theta \equiv 0$, we considered the problem (1), (2) in the work [5–7]. In the linear case, i. e., $f(t, x, z) = f(t, x)$, the problem (1), (2) was studied in the articles [8; 9]. Similar problems were also solved for other linear parabolic and hyperbolic equations [2; 3; 8–14].

Constructing the solution of the Cauchy problem. Introduce into consideration the operator K acting by the formula

$$K[u](t, x) = \frac{\varphi(x - at) + \varphi(x + at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \psi(\xi) d\xi +$$

$$+ \frac{1}{2a}\int_0^t d\tau \int_{x-a(t-\tau)}^{x+a(t-\tau)} (\Theta(\tau, \xi)u(t_0, x_0) + f(\tau, \xi, u(\tau, \xi)))d\xi, \quad (t, x) \in \overline{Q}.$$

In the closure $\overline{Q}$ of the domain Q we consider the nonlinear integral equation

$$u(t, x) = K[u](t, x), \quad (t, x) \in \overline{Q}. \tag{3}$$

T h e o r e m 1. *Let the conditions* $\Theta \in C^1(\overline{Q})$, $f \in C^1(\overline{Q} \times \mathbb{R})$, $\varphi \in C^2(\mathbb{R})$, *and* $\psi \in C^1(\mathbb{R})$ *be satisfied. The function u belongs to the class* $C^2(\overline{Q})$ *and satisfies Eq. (1) and the Cauchy conditions (2) if and only if it is a continuous-differentiable solution of Eq. (3).*

The proof can be carried out similarly to [6].

R e m a r k 1. *The condition* $\Theta \in C^1(\overline{Q})$ *of Theorem 1 can be replaced with*

$$\Theta \in C(\overline{Q}), \quad \left(\overline{Q} \ni (t, x) \mapsto \int_0^t \Theta(\tau, x \pm a(t - \tau))d\tau \in \mathbb{R}\right) \in C(\overline{Q}).$$

R e m a r k 2. *If the function* Θ *has the form* $\Theta(t, x) = \Theta(t)$ *or* $\Theta(t, x) = \Theta(x)$, *then the condition* $\Theta \in C^1(\overline{Q})$ *of Theorem 1 can be replaced with* $\Theta \in C(\overline{Q})$.

The proof of Remarks 1 and 2 follows from [15, p. 142–143].

Let us introduce the set

$$\Delta(t_P, x_P) = \{(t, x) : 0 \le t \le t_P \wedge |x - x_P| \le a|t - t_P|\}, \quad t_P \in \mathbb{R}, \quad x_P \in (0, \infty),$$

and examine some properties of the operator K. Firstly, it is obvious that the operator

$$K : C(\Delta(t_P, x_P)) \mapsto C(\Delta(t_P, x_P))$$

is well defined if $(t_0, x_0) \in \Delta(t_P, x_P)$. Secondly, there is an estimate

$$\|K[u_1] - K[u_2]\|_{C(\Delta(t_P, x_P))} \le \frac{\left(\|L\|_{C(\Delta(t_P, x_P))} + \|\Theta\|_{C(\Delta(t_P, x_P))}\right)T^2 \|u_1 - u_2\|_{C(\Delta(t_P, x_P))}}{2} \tag{4}$$

provided that the function f satisfies the Lipschitz condition with bounded function in the third variable

$$L : \Delta(t_P, x_P) \ni (t, x) \mapsto L(t, x) \in [0, \infty),$$

i. e.,

$$|f(t, x, z_1) - f(t, x, z_2)| \le L(t, x)|z_1 - z_2|. \tag{5}$$

Hence, from Banach's fixed point theorem, the obtained estimate (4) implies the solvability of Eq. (3) in the space $C(\Delta(t_P, x_P))$ provided the Lipschitz condition (5) and $\left(\|L\|_{C(\Delta(t_P, x_P))} + \|\Theta\|_{C(\Delta(t_P, x_P))}\right)T^2 < 2.$ Thus, if the following inequality holds

$$t_0 < \sqrt{2\left(\|L\|_{C(\Delta(t_0, x_0))} + \|\Theta\|_{C(\Delta(t_0, x_0))}\right)^{-1}}, \tag{6}$$

we can construct a unique solution of Eq. (3) in the space $C(\Delta(t_0, x_0))$. And, therefore, we define the value $u_0 = u(t_0, x_0)$ in a unique way. After this, the original problem (1), (2) is reduced to the Cauchy problem for the telegraph equation with a nonlinear potential

$$\Box u(t, x) - \Theta(t, x)u_0 = f(t, x, u(t, x)), \quad (t, x) \in Q, \tag{7}$$

with the initial conditions (2). The solution to the problem (7), (2) is known [6; 7]; it exists and is unique under smoothness conditions[1]

$$\begin{gathered} f \in C^1(\bar{Q} \times \mathbb{R}), \ \varphi \in C^2(\mathbb{R}), \ \psi \in C^1(\mathbb{R}), \\ \Theta \in C(\bar{Q}), \ \left(\bar{Q} \ni (t, x) \mapsto \int_0^t \Theta(\tau, x \pm a(t - \tau))d\tau \in \mathbb{R}\right) \in C(\bar{Q}), \end{gathered} \tag{8}$$

and the Lipschitz condition (5), where $L \in L^2_{\mathrm{loc}}(\bar{Q})$. But the question arises: "Do the solution to the problem (7), (2) solve the problem (1), (2)?". The following assertion answers to this question.

A s s e r t i o n. *Consider two coupled solvable equations*

$$\Box v(t, x) - \Theta(t, x)v(t_0, x_0) = f(t, x, v(t, x)), \quad (t, x) \in Q, \tag{9}$$

$$\Box u(t, x) - \Theta(t, x)v(t_0, x_0) = f(t, x, u(t, x)), \quad (t, x) \in Q,$$

with the initial conditions

$$u(0, x) = v(0, x) = \varphi(x), \ \partial_t u(0, x) = \partial_t v(0, x) = \psi(x), \ x \in \mathbb{R}.$$

Let the smoothness conditions (8) be satisfied. Then $u \equiv v$.

P r o o f. Let $w = u - v$. Then we have

$$\Box w(t, x) = f(t, x, u(t, x)) - f(t, x, v(t, x)), \quad (t, x) \in Q, \tag{10}$$

and

$$w(0, x) = \partial_t w(0, x) = 0, \quad x \in \mathbb{R}. \tag{11}$$

Using the mean value theorem, we can rewrite Eq. (10) as

$$\Box w(t, x) = \lambda(t, x)w(t, x), \quad (t, x) \in Q, \tag{12}$$

where

$$\lambda(t, x) = \int_0^1 \partial_z f(t, x, z = \xi u(t, x) + (1 - \xi)v(t, x))d\xi.$$

It is known [16] that the solution to the linear problem (11), (12) is unique. Hence $w \equiv 0$. It implies $u \equiv v$. The assertion is proved.

Thus, we have obtained a solution to the Cauchy problem (1), (2). We state the result as the following assertion.

T h e o r e m 2. *Let the smoothness conditions (8), the Lipshitz condition (5), where* $L \in L^2_{\mathrm{loc}}(\bar{Q})$, *and the smallness condition of the quantity* t_0 *(6) be satisfied*[2]. *The Cauchy problem has a unique solution u in the class* $C^2(\bar{Q})$.

The proof follows from Theorem 1 and the above argument.

[1] Existence and uniqueness theorems of the articles [6; 7] require continuous differentiability of the function Θ, but this condition can be weakened like Remarks 1 and 2 of the present communication.

[2] We do not explicitly note that the condition (6) also requires the finiteness of the quantity $\sup\limits_{(t, x) \in \Delta(t_0, x_0)} |L(t, x)|$.

R e m a r k 3. *The smallness condition (6) of Theorem 2 can be weakened to*

$$\iint\limits_{\Delta(t_0,x_0)} \left(|\Theta(\tau,\xi)| + |L(\tau,\xi)|\right) d\tau d\xi < 2a. \tag{13}$$

The proof follows from the fact that the inequality (9) is a more precise criterion for the fact that the mapping $K : C(\Delta(t_0, x_0)) \mapsto C(\Delta(t_0, x_0))$ is contraction.

Now, we will try to weaken the condition (13). We rewrite Eq. (3) in the form

$$B[u](t,x) = G(t,x), \quad (t,x) \in \overline{Q},$$

where

$$G(t,x) = \frac{\varphi(x-at)+\varphi(x+at)}{2} + \frac{1}{2a}\int\limits_{x-at}^{x+at} \psi(\xi)d\xi, \; B[u](t,x) = A[u](t,x) + A_1[u](t,x),$$

$$A[u](t,x) = u(t,x) - \frac{1}{2a}\int\limits_0^t d\tau \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} f(\tau,\xi,u(\tau,\xi))d\xi, \; A_1[u](t,x) = -\frac{1}{2a}\int\limits_0^t d\tau \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} \Theta(\tau,\xi)u(t_0,x_0)d\xi.$$

The operators $A : C(\Delta(t_0, x_0)) \mapsto C(\Delta(t_0, x_0))$, $A_1 : C(\Delta(t_0, x_0)) \mapsto C(\Delta(t_0, x_0))$ and $B : C(\Delta(t_0, x_0)) \mapsto C(\Delta(t_0, x_0))$ are Lipschitz continuous, provided that the function f satisfies the Lipschitz condition (5). The operator A_1 is has a Lipschitz constant

$$\mathcal{L}_{B-A} = \frac{1}{2a}\iint\limits_{\Delta(t_0,x_0)} |\Theta(\tau,\xi)| d\tau d\xi.$$

Since the operator A is continuously invertible [7], the operator B is continuously invertible if $\mathcal{L}_{B-A} < \gamma$ [17], where

$$\gamma = \inf_{u_1 \neq u_2} \frac{\|A[u_1] - A[u_2]\|_{C_1(\Delta(t_0,x_0))}}{\|u_1 - u_2\|_{C_1(\Delta(t_0,x_0))}}.$$

The value of γ can be obtained as $\gamma = c^{-1}$ if an a priori estimate of the form

$$\|u\|_{C_1(\Delta(t_0,x_0))} \le c\|G\|_{C_1(\Delta(t_0,x_0))}$$

for the equation

$$A[u](t,x) = G(t,x), \quad (t,x) \in \overline{Q} \tag{14}$$

is known. Let $f(t,x,0) = 0.$ Then for the solution u of Eq. (14) we have

$$|u(t,x)| \le |G(t,x)| + \frac{1}{2a}\int\limits_0^t d\tau \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} |L(\tau,\xi)||u(t,x)|d\xi, \quad (t,x) \in \overline{Q}.$$

After applying the multidimensional Grönwall lemma [18] and taking the supremum, we get

$$\|u\|_{C_1(\Delta(t_0,x_0))} \le \|G\|_{C_1(\Delta(t_0,x_0))} \exp\left(\frac{\|L\|_{L_1(\Delta(t_0,x_0))}}{2a}\right).$$

So, $\gamma = \exp\left(-(2a)^{-1}\|L\|_{L_1(\Delta(t_0,x_0))}\right)$, and the inequality $\mathcal{L}_{B-A} < \gamma$ has the form

$$\frac{1}{2a}\exp\left(\frac{1}{2a}\iint\limits_{\Delta(t_0,x_0)} |L(\tau,\xi)| d\tau d\xi\right) \iint\limits_{\Delta(t_0,x_0)} |\Theta(\tau,\xi)| d\tau d\xi \le 1.$$

We note that the same result can be also obtained using the method of continuation with respect to a parameter. We state the result as the following assertion.

T h e o r e m 3. *Let the smoothness conditions (8), the Lipshitz condition (5), where* $L \in L^2_{\text{loc}}(\overline{Q})$, *the trace condition* $f(t, x, 0) = 0$, *and the smallness condition of the quantity* t_0 *(14) be satisfied. The Cauchy problem has a unique solution u in the class* $C^2(\overline{Q})$.

Let us consider the following example.

E x a m p l e. *Let us find the solution of the problem (1), (2) in the following case*

$$f(t, x, z) = z, \ \varphi(x) = x, \ \psi(x) = 0, \ \Theta(t, x) = 1, \ t_0 = \frac{1}{4}, \ x_0 = 1, \ a = 1. \tag{15}$$

Following our developed theory, we will look for a solution using the method of successive approximations. Take the initial approximation $u_0(t, x) = 0.$ Then every subsequent approximation will be calculated by the formula

$$u_m(t, x) = x + \frac{1}{2}\int_0^t d\tau \int_{x-t+\tau}^{x+t-\tau} \left(u_{m-1}\left(\frac{1}{4}, 1\right) + u_{m-1}(\tau, \xi) \right) d\xi, \quad m \in \mathbb{N}.$$

We compute

$$u_1(t, x) = x, \ u_2(t, x) = \frac{t^2}{2} + x + \frac{t^2 x}{2}, \ u_3(t, x) = \frac{17t^2}{32} + \frac{t^4}{24} + x + \frac{t^2 x}{2} + \frac{t^4 x}{24},$$

$$u_4(t, x) = \frac{32711t^2}{6144} + \frac{17t^4}{384} + \frac{t^6}{720} + x + \frac{t^2 x}{2} + \frac{t^4 x}{24} + \frac{t^6 x}{720},$$

$$u_5(t, x) = \frac{15702011t^2}{2949120} + \frac{32711t^4}{73728} + \frac{17t^6}{11520} + \frac{t^8}{40320} + x + \frac{t^2 x}{2} + \frac{t^4 x}{24} + \frac{t^6 x}{720} + \frac{t^8 x}{40320}.$$

We can continue the calculations to find a closer approximation. However, note that the functions u_m can be represented in the form $u_m(t, x) = U_m(t) + xC_m(t)$, where the function C_m coincides with approximations of the hyperbolic cosine by the Maclaurin series. Therefore, we look for a solution to the problem (1), (2), and (15) having the form

$$u(t, x) = U(t) + x\,\text{ch}(t). \tag{16}$$

Substituting (16) into Eq. (1) and the Cauchy conditions (2), we obtain the Cauchy problem for the loaded ordinary differential equation

$$U''(t) - U(t) - U\left(\frac{1}{4}\right) = \text{ch}\left(\frac{1}{4}\right), \ U'(0) = U(0) = 0. \tag{17}$$

We can solve (17) and obtain

$$U(t) = \frac{(1 + \exp(1/2))(1 - \text{ch}(t))}{1 - 4\exp(1/4) + \exp(1/2)}. \tag{18}$$

Thus, we have found the solution of the problem (1), (2) and (15), represented in the explicit analytical form (16), (18). It is unique due to Theorem 2.

Non-uniqueness of the solution. Let us show that under some conditions the problem (1), (2) has infinitely many global classical solutions. Indeed, let

$$f(t, x, z) = |z|^{\alpha}, \ 0 < \alpha < 1, \ \varphi \equiv 0, \ \psi \equiv 0, \ \Theta(t, x) \equiv \Theta = \text{const}. \tag{19}$$

It is oblivious that that the problem (1), (2), (19) has the trivial solution $u \equiv 0$. Let us find nontrivial solutions. Note that the problem (1), (2), (19) with $\Theta = 0$, i. e., without the term $\Theta(t, x)\delta_{(t_0, x_0)}[u](t, x)$, has a one-parameter family of solutions

$$u_{p;s}(t, x) = \begin{cases} 0, & t \in [0, s), \\ u_p(t - s, x), & t \in [s, +\infty), \end{cases} \tag{20}$$

with parameter $s > 0$ [19], where

$$u_p(t, x) = 2^{\frac{1}{\alpha-1}} \left(\frac{\alpha+1}{\alpha^2 - 2\alpha + 1} \right)^{\frac{1}{\alpha-1}} t^{\frac{2}{1-\alpha}}.$$

It means that the problem (1), (2), (19) has the one-parameter family of solutions (20) with parameter $s > t_0$.

It should be noted that in this example (1), (2), (19) the function *f* does not satisfy the Lipschitz condition and is not differentiable everywhere on the set of real numbers with respect to the third variable.

Conclusions. In the present paper, we have obtained the necessary and sufficient conditions under which there exists a unique classical solution of the initial value problem for the semilinear wave equation with a Dirac potential. And we have proposed an approach to constructing solutions for equations with Dirac potentials, even for nonlinear ones.

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