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TUNNELING THROUGH A SMOOTH PARABOLIC DOUBLE BARRIER

(Communicated by Corresponding Member Lev M. Tomilchik)

Abstract. The exact description of tunneling is given for a smooth symmetric double barrier which is constructed with the help of both parabolic and inverted parabolic potentials. The analytical expression for transmission coefficient is found. The resonant tunneling condition is obtained. The dependence of transmission coefficient on incident particle energy is presented for different values of double barrier parameters. It is established that the number of resonances increases with growing the width of barriers and the distance between barriers. The continuous wave functions are expressed in terms of the confluent hypergeometric functions. The real and imaginary components of wave functions are shown at the resonance values of energy. The proposed smooth parabolic potential extends a very limited list of exactly solvable models that describe tunneling through double barriers. The variable shape of the considered double barrier gives the supplementary possibilities to simulate tunneling processes.

Keywords: tunneling, parabolic double barrier, transmission coefficient

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ТУННЕЛИРОВАНИЕ ЧЕРЕЗ ГЛАДКИЙ ПАРАБОЛИЧЕСКИЙ ДВОЙНОЙ БАРЬЕР

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Аннотация. Дано точное описание туннелирования для гладкого симметричного двойного барьера, который построен с помощью как параболических, так и перевернутых параболических потенциалов. Найдено аналитическое выражение для коэффициента прохождения. Получено условие резонансного туннелирования. Представлена зависимость коэффициента прохождения от энергии налетающей частицы для различных значений параметров двойного барьера. Установлено, что число резонансов растет с увеличением ширины барьеров и расстояния между барьерами. Непрерывные волновые функции выражены через вырожденные гипергеометрические функции. Показаны реальные и мнимые составляющие волновых функций при резонансных значениях энергии. Предложенный параболический потенциал расширяет весьма ограниченный перечень точно решаемых моделей, которые описывают туннелирование через двойные барьеры. Варьируемая форма рассматриваемого двойного барьера дает дополнительные возможности моделирования процессов туннелирования.

Ключевые слова: туннелирование, параболический двойной барьер, коэффициент прохождения

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Introduction. The first observation of the resonant tunneling in semiconductor geterostructures [1] induced considerable interest to a wide variety of potentials which can simulate the double-barrier physical structures. For instance, the rectangular [2], triangular [3] and trapezoidal [4] double-barrier potentials were considered. These potentials are not smooth but allow the exact solutions of the Schrödinger equation. The smooth potential was proposed in [5] using Gaussian functions, however this potential does not permit exact analytical solution. The phenomenon of resonant tunneling was also analyzed in the framework of model with the parabolic well between two rectangular barriers [6]. At last, the double-barrier potential was composed with the help of two separated inverted parabolas in [7]. Note that the first derivatives of potentials in [6] and [7] are discontinuous. At the same time the smooth single barrier was constructed in [8] using both parabolas and inverted parabolas. It is not hard to perform transition from the single barrier to the double barrier by means of the simple duplication of the potential profile proposed in [8].

The new symmetric potential function is of the form

$$V(q)=V_0\begin{cases}0, & (2+k)q_0<|q|,\\ \dfrac{(|q|-(2+k)q_0)^2}{(1-g)q_0^2}, & (1+g+k)q_0<|q|<(2+k)q_0,\\ 1-\dfrac{(|q|-(1+k)q_0)^2}{gq_0^2}, & (1-g+k)q_0<|q|<(1+g+k)q_0,\\ \dfrac{(|q|-kq_0)^2}{(1-g)q_0^2}, & kq_0<|q|<(1-g+k)q_0,\\ 0, & 0<|q|<kq_0.\end{cases} \quad (1)$$

Here $0<g<1$, $k>0$, $2q_0$ is the width of each barrier and $2kq_0$ is the distance between barriers. The second derivative of the function (1) is discontinuous at the points $\mp(2+k)q_0$, $\mp(1+g+k)q_0$, $\mp(1-g+k)q_0$ and $\mp kq_0$. However, both the function (1) and its first derivative are continuous. The presence of a varied parameter g allows to change a shape of double-barrier potential in the wide range.

Analytical description of tunneling. We are interesting in solving the Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dq^2}+V(q)\right)\Psi(q)=E\Psi(q),$$

where $V(q)$ takes the form (1). It is convenient to introduce dimensionless quantities

$$x=\sqrt{\frac{2mV_0}{\hbar^2}}\,q,\quad x_0=\sqrt{\frac{2mV_0}{\hbar^2}}\,q_0,\quad e=\frac{E}{V_0}.$$

The transformed Schrödinger equation is given as

$$\left(-\frac{d^2}{dx^2}+v(x)\right)\psi(x)=e\psi(x) \quad (2)$$

with the scaled potential

$$v(x)=\begin{cases}0, & (2+k)x_0<|x|,\\ \dfrac{(|x|-(2+k)x_0)^2}{(1-g)x_0^2}, & (1+g+k)x_0<|x|<(2+k)x_0,\\ 1-\dfrac{(|x|-(1+k)x_0)^2}{gx_0^2}, & (1-g+k)x_0<|x|<(1+g+k)x_0,\\ \dfrac{(|x|-kx_0)^2}{(1-g)x_0^2}, & kx_0<|x|<(1-g+k)x_0,\\ 0, & 0<|x|<kx_0.\end{cases} \quad (3)$$

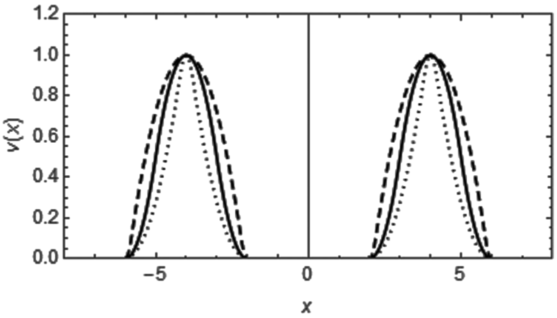


Fig. 1. The scaled potential $v(x)$ for different values of g

The shape of $v(x)$ is shown in fig. 1 for different values of g when $x_0 = 2$ and $k = 1$. Here and in subsequent figures we use dotted lines for $g = 0.1$, solid lines for $g = 0.5$ and dashed lines for $g = 0.9$.

The simplicity of the considered potential (3) permits to find the exact solutions of Eq. (2) in nine regions. The wave function is represented in the following way

$$\psi(x) = \begin{cases} \exp(i\sqrt{e}\,x) + A_1 \exp(-i\sqrt{e}\,x), & x < -(2+k)x_0, \\ A_2\ y_{s1}(z_s^{++}) + A_3\ y_{s2}(z_s^{++}), & -(2+k)x_0 < x < -(1+g+k)x_0, \\ A_4\ y_{c1}(z_c^{+}) + A_5\ y_{c2}(z_c^{+}), & -(1+g+k)x_0 < x < -(1-g+k)x_0, \\ A_6\ y_{s1}(z_s^{+}) + A_7\ y_{s2}(z_s^{+}), & -(1-g+k)x_0 < x < -kx_0, \\ A_8 \cos(\sqrt{e}\,x) + A_9 \sin(\sqrt{e}\,x), & -kx_0 < x < kx_0, \\ A_{10}\ y_{s1}(z_s^{-}) + A_{11}\ y_{s2}(z_s^{-}), & kx_0 < x < (1-g+k)x_0, \\ A_{12}\ y_{c1}(z_c^{-}) + A_{13})\ y_{c2}(z_c^{-}), & (1-g+k)x_0 < x < (1+g+k)x_0, \\ A_{14}\ y_{s1}(z_s^{--}) + A_{15}\ y_{s2}(z_s^{--}), & (1+g+k)x_0 < x < (2+k)x_0, \\ A_{16} \exp(i\sqrt{e}x), & (2+k)x_0 < x. \end{cases}$$

There are the incident and reflected waves in the region $x < -(2+k)x_0$ and there is the transmitted wave in the region $x > (2+k)x_0$. It is not hard to show that the particular solutions in the regions $kx_0 < |x| < (2+k)x_0$ are expressed in terms of the confluent hypergeometric functions [9]. In the regions $(1+g+k)x_0 < |x| < (2+k)x_0$ and $kx_0 < |x| < (1-g+k)x_0$, the explicit solutions are given by formulas

$$y_{s1}(z_s) = e^{-z_s^2/4} M\left(\frac{a_s}{2} + \frac{1}{4}, \frac{1}{2}, \frac{z_s^2}{2}\right),$$

$$y_{s2}(z_s) = z_s e^{-z_s^2/4} M\left(\frac{a_s}{2} + \frac{3}{4}, \frac{3}{2}, \frac{z_s^2}{2}\right),$$

$$z_s^{\pm\pm}(x) = \left(\frac{2}{x_0}\right)^{1/2} \frac{(x \pm (2+k)x_0)}{(1-g)^{1/4}}, \quad z_s^{\pm}(x) = \left(\frac{2}{x_0}\right)^{1/2} \frac{(x \pm kx_0)}{(1-g)^{1/4}},$$

$$a_s = -\frac{\sqrt{1-g}}{2} x_0 e.$$

In the regions $(1-g+k)x_0 < |x| < (1+g+k)x_0$, we have the following solutions

$$y_{c1}(z_c) = \frac{1}{2}\left\{ e^{-iz_c^2/4} M\left(-\frac{ia_c}{2} + \frac{1}{4}, \frac{1}{2}, \frac{iz_c^2}{2}\right) + e^{iz_c^2/4} M\left(\frac{ia_c}{2} + \frac{1}{4}, \frac{1}{2}, -\frac{iz_c^2}{2}\right)\right\},$$

$$y_{c2}(z_c) = \frac{z_c}{2}\left\{ e^{-iz_c^2/4} M\left(-\frac{ia_c}{2} + \frac{3}{4}, \frac{3}{2}, \frac{iz_c^2}{2}\right) + e^{iz_c^2/4} M\left(\frac{ia_c}{2} + \frac{3}{4}, \frac{3}{2}, -\frac{iz_c^2}{2}\right)\right\},$$

$$z_c^{\pm}(x) = \left(\frac{2}{x_0}\right)^{1/2} \frac{(x \pm (1+k)x_0)}{g^{1/4}},$$

$$a_c = \frac{\sqrt{g}}{2} x_0 (1-e).$$

It should be stressed that these solutions are real.

Applying the continuity conditions on the wave function and its first derivative at eight points $\mp(2+k)x_0$, $\mp(1+g+k)x_0$, $\mp(1-g+k)x_0$ and $\mp kx_0$, one can get the system of sixteen algebraic equations for sixteen coefficients A_i. It is easily to solve this system but the solutions are very cumbersome. Therefore we represent only one coefficient

$$A_{16} = \frac{1}{2}\left(\frac{L_- + i\sqrt{e}}{L_- - i\sqrt{e}} - \frac{L_+ + i\sqrt{e}}{L_+ - i\sqrt{e}}\right) \exp(-2(2+k)ix_0\sqrt{e}),$$

where we use notations

$$L_{+}=\left(1-\frac{2\sqrt{e}}{b_1+b_2}\tan(kx_0\sqrt{e})\right)^{-1}\left(\frac{2b_1b_2}{b_1+b_2}-\sqrt{e}\tan(kx_0\sqrt{e})\right),$$

$$L_{-}=\left(1+\frac{b_1+b_2}{2\sqrt{e}}\tan(kx_0\sqrt{e})\right)^{-1}\left(\frac{b_1+b_2}{2}+\frac{b_1b_2}{\sqrt{e}}\tan(kx_0\sqrt{e})\right),$$

$$b_1=\left(\frac{2}{x_0}\right)^{1/2}\frac{1}{(1-g)^{1/4}}\frac{f_{11}}{f_{21}},\quad b_2=\left(\frac{2}{x_0}\right)^{1/2}\frac{1}{(1-g)^{1/4}}\frac{f_{12}}{f_{22}},$$

$$f_{ij}=g^{-1/4}\bar{y}_{si}\bar{y}'_{cj}+(1-g)^{-1/4}\bar{y}_{cj}\bar{y}'_{si},\quad i=1,2,\quad j=1,2,$$

$$\bar{y}_{si}=y_{si}(\bar{z}_s),\quad \bar{y}'_{si}=\frac{dy_{si}(\bar{z}_s)}{d\bar{z}_s},\quad \bar{z}_s=\sqrt{2x_0}(1-g)^{3/4},$$

$$\bar{y}_{cj}=y_{cj}(\bar{z}_c),\quad \bar{y}'_{cj}=\frac{dy_{cj}(\bar{z}_c)}{d\bar{z}_c},\quad \bar{z}_c=\sqrt{2x_0}\,g^{3/4}.$$

The square of the absolute value of A_{16} is the transmission coefficient T for the proposed double-barrier potential (1). The final exact expression is

$$T=|A_{16}|^2=\left(1+\frac{d^2}{e}\right)^{-1},$$

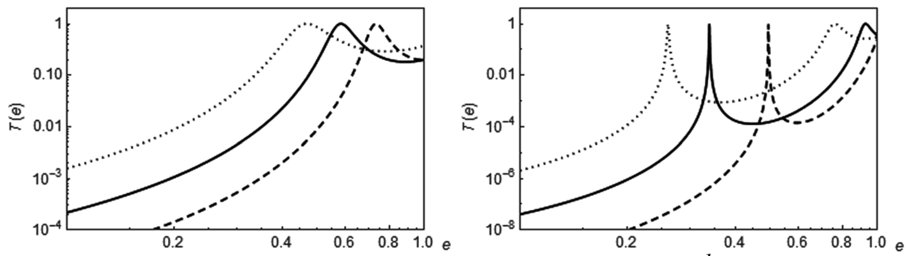
where

$$d=\frac{L_{+}L_{-}+e}{L_{+}-L_{-}}.$$

It should be noted that the resonant tunneling (T = 1) is realized at selected values of e which are the solutions of equation $d(e)=0$.

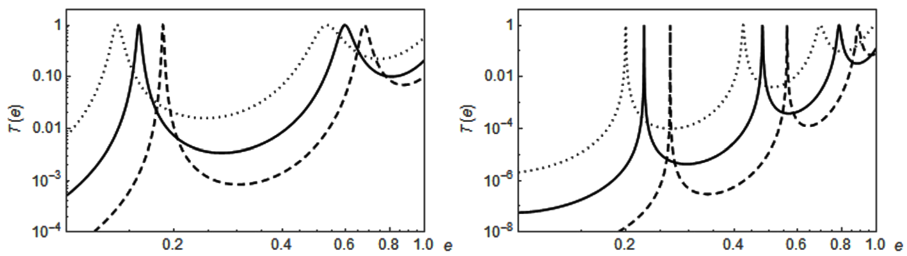
Graphic presentation of results. The dependence of the transmission coefficient T on a scaled energy e is given in figures 2–4 for different values of the barrier parameters x_0, k and g. It is seen that the number of resonances increases with the growth of x_0 and k. The resonant energies shift toward the higher values if g grows.

At last, the real (solid lines) and the imaginary (dashed lines) components of the wave functions are represented in fig. 5 at two resonant values of energy for $x_0=2$, $k=1$ and $g=0.5$.



a *b*

Fig. 2. Dependence of T on e at k = 0: a – for x_0 = 2, b – for x_0 = 4



a *b*

Fig. 3. Dependence of T on e at k = 1: a – for x_0 = 2, b – for x_0 = 4

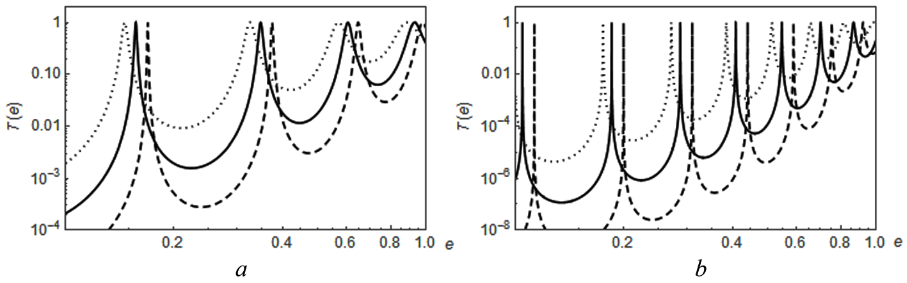


Fig. 4. Dependence of T on e at $k = 3$: a – for $x_0 = 2$, b – for $x_0 = 4$

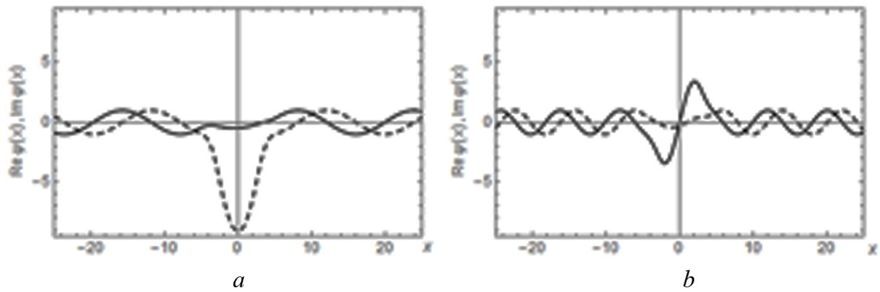


Fig. 5. Wave function for $x_0 = 2$, $k = 1$ and $g = 0.5$: a – for $e = 0.159361$, b – for $e = 0.599483$

Conclusion. The proposed smooth parabolic potential extends a very limited list of exactly solvable models that describe tunneling through double barriers. The variable shape of considered barrier gives the supplementary possibilities to simulate the tunneling processes. In addition to the symmetric potential examined in the present paper it is desirable to consider more complicated asymmetric smooth parabolic potential which will allow to find the exact solution too.

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